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Citation: J. Renewable Sustainable Energy **5**, 052013 (2013); doi: 10.1063/1.4826940
View online: http://dx.doi.org/10.1063/1.4826940
View Table of Contents: http://jrse.aip.org/resource/1/JRSEBH/v5/i5
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JOURNAL OF RENEWABLE AND SUSTAINABLE ENERGY **5**, 052013 (2013)

Optimization of a counter-rotating wind turbine using the blade element and momentum theory

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(Received 24 January 2013; accepted 11 October 2013; published online 31 October 2013)

A counter-rotating wind turbine has a front rotor and a rear rotor which rotate in opposite directions on the same axis. Compared to a single rotor, the flow field of the counter-rotating wind turbine is complicated due to the interactions between the front rotor and the rear rotor. The wake induced by the front rotor works on the inflow of the rear rotor and is essentially an unsteady flow state. In order to estimate the performance of a counter-rotating wind turbine, it is necessary to consider more variables than the case of a single-rotor wind turbine and to prepare an estimation system capable of performing delicate predictions. For the optimization of a counter-rotating wind turbine, the pitch angles, radius ratios, and rotation speeds of two rotors are chosen as the design values and variations of the power coefficients and thrust coefficients can be observed in this study. The torque balance of the two rotors due to the kinematic coupling of the generator is considered. Modeling by means of the blade element and momentum theory for the optimization of a counter-rotating wind turbine is developed to predict the front rotor flow and the wake flow generated by the front rotor. The wake flow is then applied for the inflow of the rear rotor. A vehicle test is carried out to validate the prediction. The optimized solution is found using a multi-island genetic algorithm.

[http://dx.doi.org/10.1063/1.4826940]

NOMENCLATURE

a	axial induction factor
BEMT	blade element momentum theory
VLM	vortex lattice method
C_p	power coefficient
C_T	axial force coefficient
R	rotor radius
r	radial distance
U_∞	wind velocity
ρ	air density
D	diameter of the front rotor
N_b	number of blades
σ	solidity of blade $\sigma(\mathrm{r}) = \frac{cN_b}{2\pi r}$

I. INTRODUCTION

The wind turbine industry is becoming an important area in research because the importance of renewable energy is increasingly stressed around the world. In addition, active advanced technologies pertaining to wind turbines have been carried out, as the commercial value of wind turbines is gradually rising.

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A counter-rotating wind turbine has a front rotor and a rear rotor, which rotate in opposite directions on the same axis. This type of turbine has been proposed as a new model to increase power efficiency of wind turbines.[1] According to the momentum theory by Newman,[2] the maximum efficiency of a counter-rotating wind turbine is theoretically 64%, which is higher than the 59%-maximum-efficiency of the single rotor design.[3,4] In order to validate this theory, several experimental studies on counter-rotating wind turbines have been carried out.[5–7]

Counter-rotating wind turbines can be classified into various types according to the radius ratio between the front and rear rotors, the positioning of the nacelle and rotors, and the type of generator used. In this study, the target of the research is a small counter-rotating wind turbine with a front rotor that faces upwind and a rear rotor that faces downwind, with the rear rotor longer than the front rotor. This type of wind turbine has properties of the downwind rotor which is able to be a free yawing because the radius of the rear rotor is longer. Therefore, it is suitable for a small wind turbine, which cannot easily use a yawing system.

There are three types of generator configurations which can be used in counter-rotating wind turbines.[8] These are listed below.

A. Two independent generators, one for each rotor.
B. A differential planetary gearing coupling both rotors to a single generator.
C. One generator with kinematic coupling of two rotors.

When generator configurations B and C are used, the generator systems can be simplified because there is no need for a second generator. Thus, the cost of the wind turbine is reduced. In case B, high generator efficiency can be obtained as there are additional degrees of freedom. However, the maintenance cost may also increase due to a gear wear. In contrast, case C does not require a gear, making the system simpler than that of case B. This also reduces the cost of the wind turbine systems.

However, the front and rear rotors always rotate with the same shaft torque due to the kinematic coupling between the stator and the armature. As a result, the torque balance must be considered during the design process. Each rotor rotates at a different rotational speed from the design specifications during operation. If the torque balance is not properly considered, it is possible that at least one rotor will not rotate well.[9] On the other hand, it is possible that a low-efficiency design will be derived when only the torque balance is considered during the design process.

In this study, an optimization process to obtain the maximum power from a counter-rotating wind turbine is carried out while considering the torque balance. The prediction method for the optimization is the blade element and momentum theory (BEMT) with wake modeling applied. A validation process is carried out by means of a wind turbine test using vehicle.

II. METHODOLOGY

A. Numerical method

The BEMT is a hybrid method which combines momentum theory and blade element theory for an analysis of the aerodynamic performance.[10] Of each section, Momentum theory is the control volume theory that applies Plandtl's tip-loss function, and blade element theory is summation of the sectional thrust and torque as calculated by the sectional lift and drag coefficient of the airfoils. The BEMT is reliable because it is based on solid physical principles. Furthermore, the computation cost of the BEMT is remarkably low, making it suitable for an optimization process which requires a number of calculations.

The thrust and the torque according to momentum theory are as follows:

$$\mathrm{d}T_f = 4F\rho U_\infty^2 a_f(1-a_f)\pi r dr, \tag{1}$$

$$\mathrm{d}Q_f = 4F\rho U_\infty a_f'(1-a_f)\pi r^3\Omega_f dr. \tag{2}$$

The thrust and the torque according to blade element theory are as follows:

$$\mathrm{d}T_f = N_b \rho U_{rel}^2 (c_l \cos\phi + c_d \sin\phi) r dr/2, \tag{3}$$

$$\mathrm{d}Q_f = N_b \rho U_{rel}^2 (c_l \sin\phi - c_d \cos\phi) r dr/2. \tag{4}$$

The above functions are used for determining the aerodynamic performance of the front rotor.

First, the axial induction factor a_f and angular induction factor a_f' are determined using the equal part of the thrust and torque functions. Second, the thrust and the torque values are determined by replacing a_f and a_f' again.

In order to increase the accuracy of this method, the stall delay model and Glauert's tip loss model and a correction of thrust factors are added.

One of the assumptions of the BEMT is neglect of flow in the radial direction. This assumption leads to a discrepancy between a real case and a two dimensional case especially near the root of the blade, because the BEMT usually comes from two dimensional data which are wind tunnel tests or computational methods. Therefore, a stall delay model can be corrections of two dimensional data which have lift and drag coefficients. In this study, the stall delay model applied is proposed by Du and Selig.[11]

Tip loss model is applied for considering the influence of the vortex shed from blade tips at the induced velocity field

$$\mathrm{F} = \frac{2}{\pi} \cos^{-1}(e^{-f}), \tag{5}$$

where

$$\mathrm{f} = \frac{N_b}{2} \frac{R - r}{r \sin\phi}, \tag{6}$$

where F is a correction factor derived by Prandtl and r is a radius at a specific location.

The BEMT becomes invalid and the three dimensional effects cannot be ignored at the induction factor is greater than about 0.4. For this reason, Glauert developed a correction of thrust coefficient to compensate for this effect.

The aerodynamic performance of the rear rotor is also predicted using the BEMT method. To analyze the inflow condition of the rear rotor, it is assumed that the rear rotor is positioned in the fully developed stream tube of the front rotor,[12]

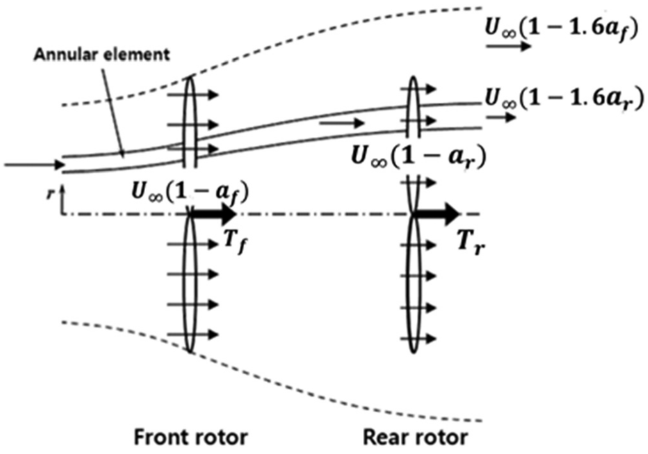

FIG. 1. Flow model for BEMT analysis of counter rotating wind turbine corrected by VLM.

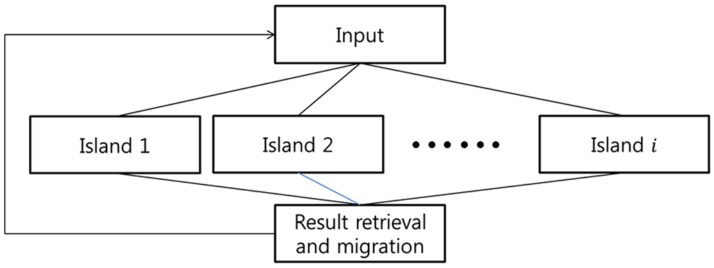

FIG. 2. Multi-island genetic algorithm.

$$U_{\infty,r} = U_{\infty}(1 - 2a_f). \tag{7}$$

However, the distance between the two rotors on the counter-rotating wind turbine designed in this study is 0.33D. Therefore, it is difficult to confirm that the rear rotor is in the fully developed stream tube. Regarding this assumption, it is possible that the aerodynamic performance is under-predicted. To address this issue, the BEMT method is corrected by the inflow condition of the Vortex Lattice Method (VLM),[13] as shown in Fig. 1 and Eq. (8),

$$U_{\infty,r} = U_{\infty}(1 - 1.6a_f). \tag{8}$$

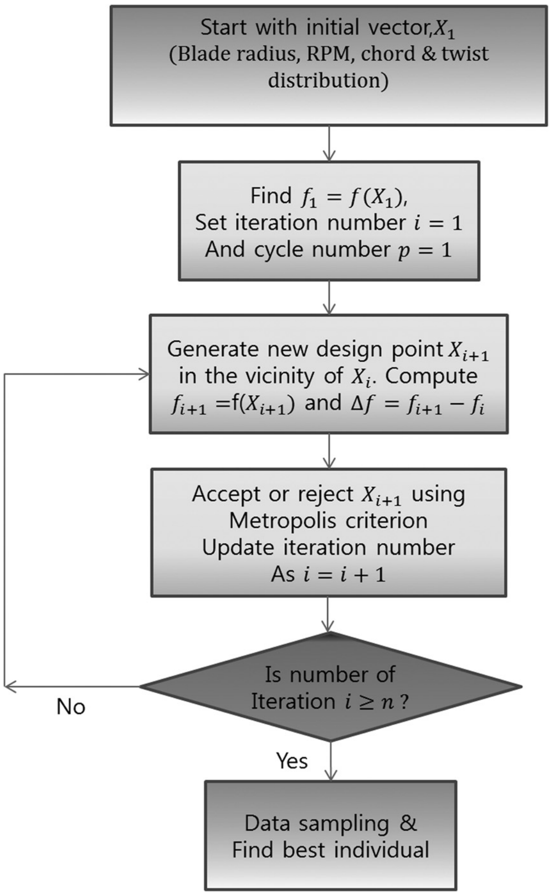

FIG. 3. Flow chart of optimization.

Because the VLM is not suitable as an optimization process due to its high computing cost, the BEMT method is used for optimization.

Based on the mass-conserving boundary condition, the degree of expansion, which is influenced by the wake of the front rotor at the rear rotor, is shown below:

$$R_e = R_f\sqrt{(1-a_f)/(1-1.6a_f)}. \tag{9}$$

The factor 1.6(value x) is approximately calculated using the relation between radii of actuator disk and induction factor. The radius of actuator disk at the rear rotor (R_x) is determined by VLM method developed by Lee *et al.*,[13]

$$\frac{R_x}{R_f} = \frac{\sqrt{1-a_f}}{\sqrt{1-xa_f}}. \tag{10}$$

B. Optimizing method

For the optimization, a multi-island genetic algorithm (MIGA) is used. The MIGA divides population into several islands, after which traditional genetic operations are performed on each

FIG. 4. Experimental equipment of wind turbine test.

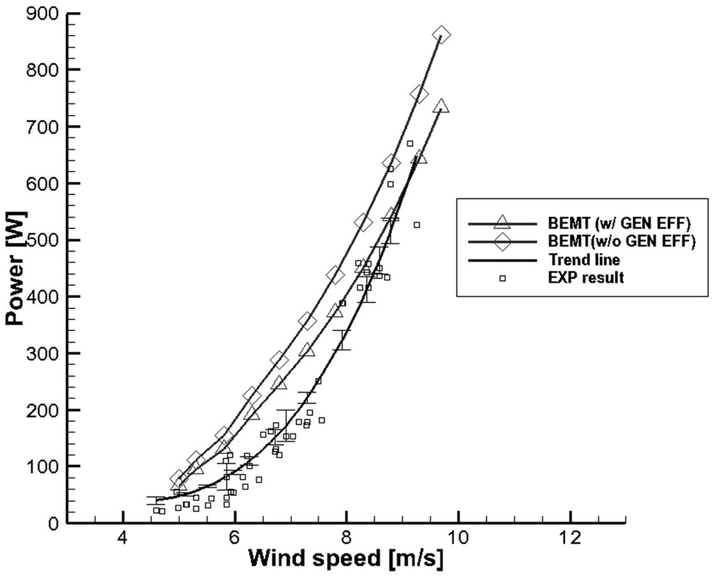

FIG. 5. Power curve of vehicle test.

island, as shown in Figs. 2 and 3. Any individual case between the islands is migrated to find a global solution. This method has a low possibility of a deriving a local solution.[14,15]

To optimize the maximum efficiency of the power, the chord length, and the twist of the two rotors, the rotational speed of the two rotors and the radius of the front rotor are selected as design values. For simultaneous rotation of two rotors, the solidity ratio of the front and the rear rotors and the torque equilibrium are selected as constraints.

III. RESULT AND DISCUSSION

The equipment of the vehicle test is shown in Fig. 4. An anemometer is installed at a distance of one rear rotor radius from the hub. The sampling frequency of the vehicle test is 1 s, with mean values averaged over 10 s used.

The validation of the BEMT was carried out by means of a vehicle test, as shown in Fig. 5. Because the generating efficiency is not considered in this result, the triangle symbol line was obtained by multiplying the BEMT result by the generating efficiency at the rated wind speed. The generating power is over-predicted in region of low wind speeds because information pertaining to the generating efficiency does not exist in these regions. The generating efficiency at the rated wind speed is used at in the overall region. In most cases, the generating efficiency at a low wind speed is lower than the generating efficiency at the rated wind speed. As a result, the prediction of the BEMT will be close to the experimental result. If this consideration is accepted, the BEMT shows reasonable prediction accuracy.

TABLE I. Performance of baseline.

	Front blade		Rear blade
Radius [m]	1.07		1.42
Solidity	0.0741		0.0492
Rated speed [m/s]		9	
Max Cp		0.21	

TABLE II. Performance of optimized model.

	Front blade	Rear blade
Radius [m]	1.11	1.24
Solidity	0.0792	0.0871
Rated speed [m/s]	8	
Max Cp	0.4	

To optimize the blade, the model of the vehicle test was selected as the baseline. The rotors of this model have a high pitch angle for the simultaneous rotation of each rotor. As a result, the baseline has a significantly low power coefficient and long radius of the rear rotor to recover the power loss as the solidity of the rear rotor is significantly lower than that of the front rotor, as shown in Table I. Consequently, the solidity must be considered.

The objective function is maximum power efficiency greater than 0.45 for a 600 W counter-rotating wind turbine. To obtain this efficiency and to ensure suitability for a small wind turbine, the rated wind speed is decreased from 9 to 8 m/s and the rear rotor radius is decreased from 1.42 to 1.24 m.

After the optimization process, a counter-rotating wind turbine with a maximum power efficiency of 0.47 is obtained. The generating efficiency is not considered at this result. In addition, the solidity ratio of the rear rotor, which is slightly higher than that of the front rotor, is obtained as shown in Table II.

Fig. 6 shows the power curve of the optimized model. This model is designed to generate 600 W at 8 m/s. Constraints of the torque balance are well reflected, because the torque ratio is nearly zero overall. The maximum power efficiency considering the generator efficiency is 0.4 at the rated wind speed, while the maximum power efficiency of the baseline is 0.21. Compared to the baseline, the power efficiency of this model shows a significant increase.

The chord-length distribution according to span-wise is shown in Fig. 7. The solidity difference between the front and rear rotors on baseline is relatively large. Because of an unbalanced torque state at a starting point of a rotation, this difference can make a failure of a co-rotation. In order to solve this problem, the solidity ratio between two rotors is considered. As a result, the solidity of the rear rotor is slightly larger than the front rotor's on the optimization model for co-rotation between front and rear rotors.

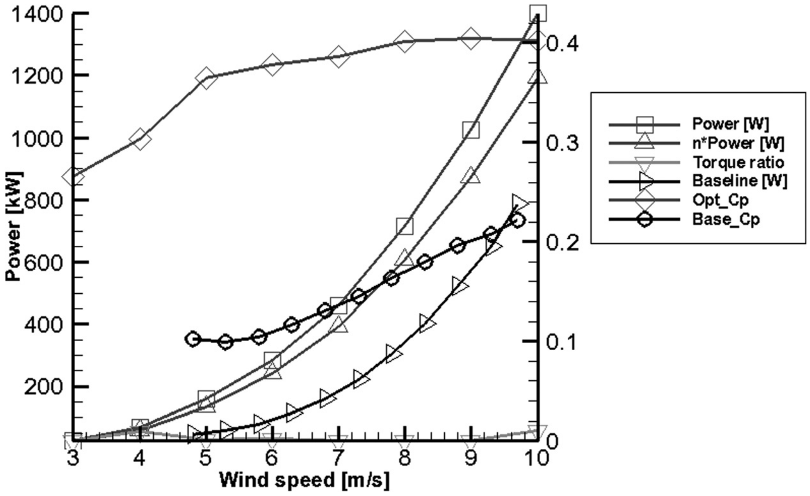

FIG. 6. Power curve for optimization.

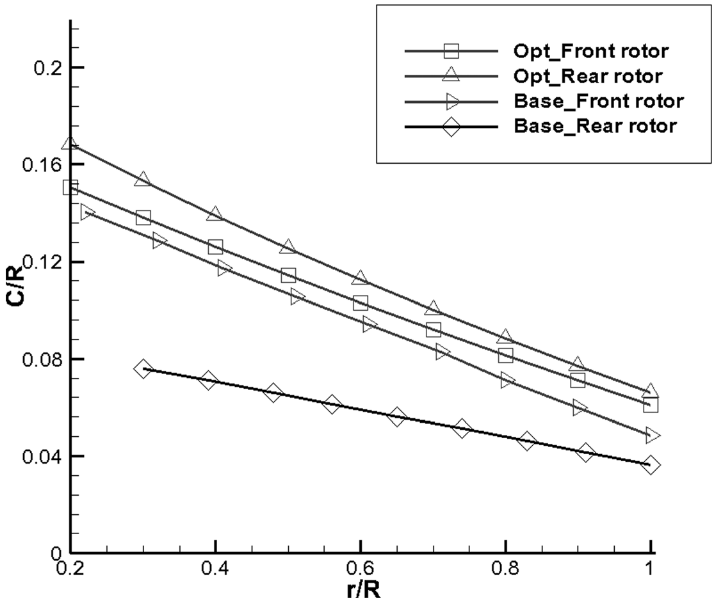

FIG. 7. Chord/Radius distribution.

The twist distribution according to span-wise is shown in Fig. 8. The functions of twist are configured by using a linear fractional function,

$$f(x) = \frac{a}{x - b} + c. \tag{11}$$

The value x is a radial position, r/R. The coefficients a, b, and c are used for design values of the optimization.

The rotational speed curve line is shown in Fig. 9. Because the reference radius which is a radius of a rear rotor is changed, the area of the rotational speed is changed. These RPM are determined by torque-equilibrium-condition.

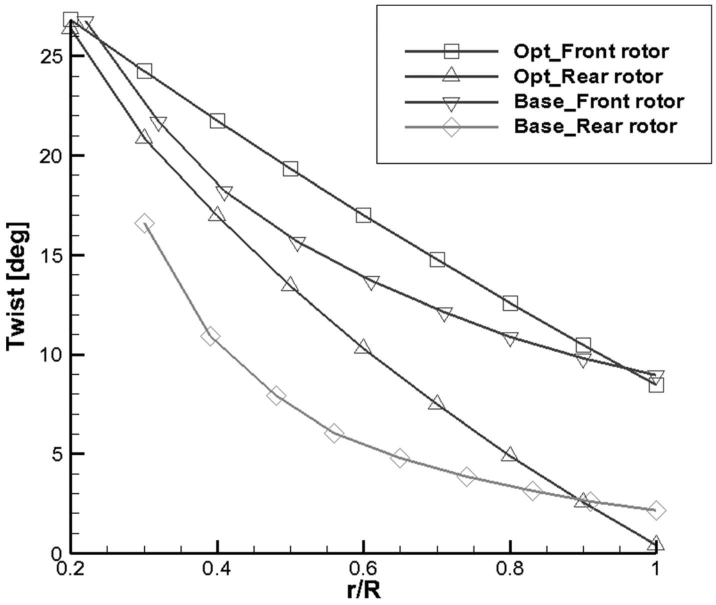

FIG. 8. Twist curve line.

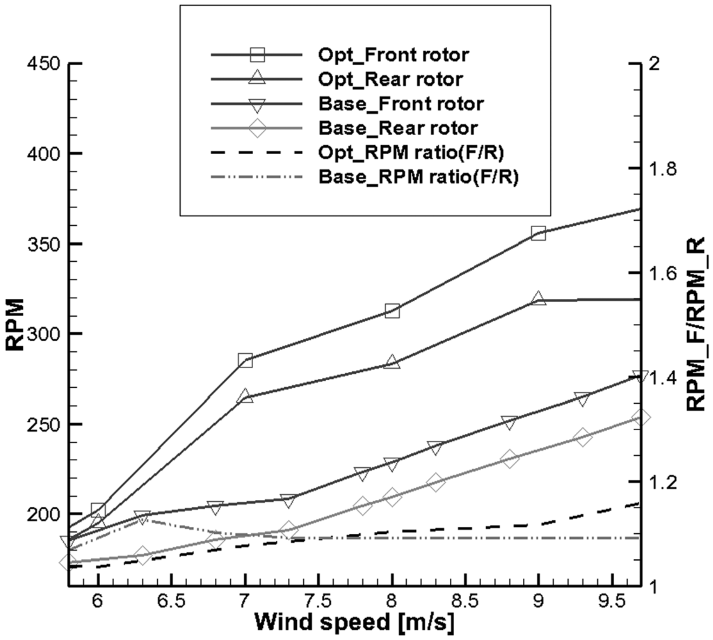

FIG. 9. RPM curve line.

In order to compare how much effect the power coefficient has on the front and rear rotors, the power coefficient of each rotor is shown in Fig. 10. In case of the baseline, the power coefficient of two rotors is nearly same. In case of the optimization, although a radius of the front rotor is shorter than the rear rotor's, the power coefficient of the front rotor is larger than the rear rotor's.

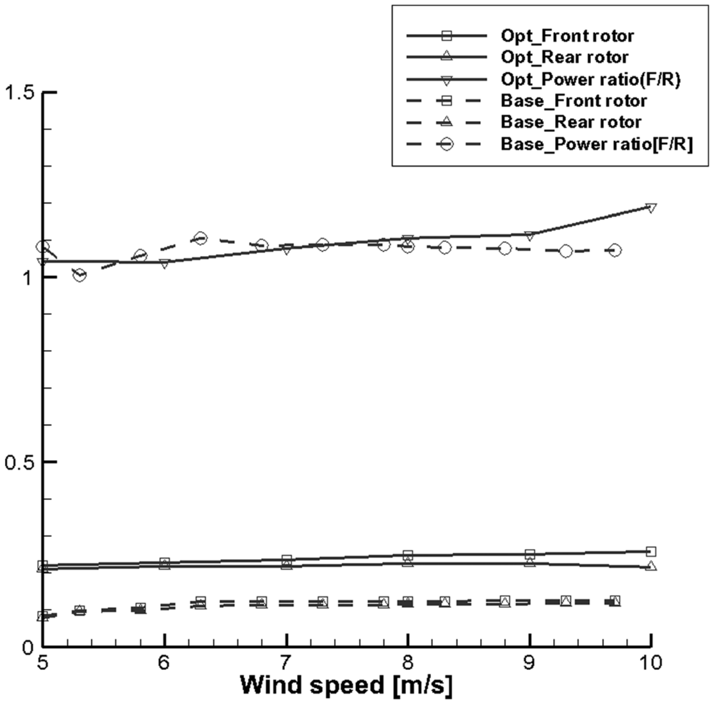

FIG. 10. Power coefficient curve line.

IV. CONCLUSION

In this study, the blade element and momentum theory were used with a counter-rotating wind turbine as means of optimization. The BEMT method was corrected by the vortex lattice method. The developed method was validated by a comparison with the power curve after a wind turbine test using a vehicle. The prediction method showed reasonable performance.

The optimization process for a counter-rotating wind turbine was established. In the process, the torque equilibrium and the solidity ratio are considered for operation involving simultaneous rotation. In addition, the rear rotor radius and the rated wind speed can be decreased, promoting the wider use of small wind turbines. The optimized counter-rotating wind turbine design was thus obtained, satisfying the objective function and constraints.

ACKNOWLEDGMENTS

This work was supported by the Human Resources Development Program (No. 20124030200030) of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Trade, Industry, and Energy.

[1] W. Z. Shen, V. A. K. Zakkam, J. N. Sorensen, and K. Appa, J. Phys.: Conf. Ser. **75**, 012003 (2007).
[2] B. G. Newman, "Actuator-disc theory for vertical-axis wind turbines," J. Wind Eng. Ind. Aerodyn. **15**, 347–355 (1983).
[3] J. F. Manwell, J. G. McGowan, and A. L. Rogers, *Wind Energy Explained* (University of Massachusetts, Amherst, USA, 2002).
[4] E. Hau, *Wind Turbines—Fundamentals, Technologies, Application, Economics*, 2nd ed. (Springer, Berlin, 2006).
[5] K. Appa, "Energy innovations small grant (EISG) program (counter rotating wind turbine system)," EISG Final Report No. 51809A/00-09 (California, US, 2002).
[6] S. Jung, T. No, and K. Ryu, Renewable Energy **30**, 631–644 (2005).
[7] J. O. Dabiri, J. Renewable Sustainable Energy **3**, 043104 (2011).
[8] R. Herzog, A. P. Schaffarczyk, A. Wachinski, and O. Zürcher, "Performance and stability of a counter-rotating windmill using a planetary gearing: Measurements and simulation." EWEC **6**(6), 4847 (2010).
[9] T. Kanemoto and A. M. Galal, JSME Int. J., Ser. B **49**(2), 450–457 (2006).
[10] J. G. Leishman, *Principles of Helicopter Aerodynamics* (Cambridge University Press, New York, 2006), pp. 727–747.
[11] Z. Du and M. Selig, "A 3-D stall-delay model for horizontal axis wind turbine performance prediction," AIAA Paper No. 98-0021, 1998.
[12] S. Lee, H. Kim, E. Son, and S. Lee, Renewable Energy **42**, 140–144 (2012).
[13] S. Lee, E. Son, and S. Lee, "Velocity interference in the rear rotor of a counter-rotating wind turbine," Renewable Energy **59**, 235–240 (2012).
[14] M. Melanie, *An Introduction to Genetic Algorithms* (Massachusetts Institute of Technology, 1996).
[15] M. Kaneko, *A Parallel Genetic Algorithm with Distributed Environment Scheme* (Doshisha University, Intelligent Systems Design Laboratory, Japan, 1999).